STRUCTURAL EQUATION MODELS WITH LATENT VARIABLES

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Structural equation models (also referred to as "SEM models") have become very popular in the Social Sciences, especially in Psychology, Sociology, Education and, more recently, in Business and Public Administration and various applied health sciences (e.g., Nursing). A major feature in the development of structural equation models from the earlier causal ("path") models of the 1960s and 1970s is the conceptualization of latent variables. The terms, "unmeasured variable models" and "latent variable models" refer to types of structural equation models that explicitly incorporate measurement error into the estimation of structural equation parameters, and treat observed ("manifest") variables as indicators of underlying constructs rather than perfectly measured representations of these same constructs. These models are quite general, and subsume many of the multivariate techniques typically dealt with in lower-level courses, including regression models, factor analysis, analysis of variance/analysis of covariance, principal components analysis, and path modeling. More recently, SEM models have provided an approach to the estimation of parameters in growth curve models for longitudinal data (these can also be estimated in the multilevel model framework), and for an approach to the problem of the unbiased estimation of parameters in the presence of missing data.

In earlier usage, the models discussed in this course were, in the 1980s and early 1990s, often referred to as LISREL models. LISREL is now but one of the many computer programs now available to estimate SEM models; there are many others, including AMOS, which is now distributed with SPSS, and MPlus, both of which we will introduce during the course.

Early in the course, we will start with a scalar presentation of latent variable models and our class/lab examples will use the the SIMPLIS (scalar) version of LISREL, scalar programming using MPlus and possibly EQS. If there is any interest on the part of class participants, some examples using other software (SAS-CALIS; GLLAMM, EQS) will be presented.

The SPSS-distributed AMOS program is very popular, in part because of its graphic interface (draw diagrams, run models, and the parameter estimates appear), and partly because large numbers of universities have SPSS site licenses (which are nonetheless quite expensive). In the past, the introduction to structural equation model software started with AMOS, but this year the use of AMOS will be optional; a special lab given outside class with an accompanying "self instruction guide" (for those who cannot attend) will cover AMOS.

After an introduction to SEM models in scalar terms, we will briefly introduce the matrix-form representation of SEM models. Much of the literature (especially earlier literature) presents models using LISREL matrix notation, and some software (e.g., LISREL) is programmed primarily in matrix form. Next, we extend the models we have learned in two overlapping directions: multiple-group models and models for means and intercepts. Towards the end of the course, we shall cover some more advanced topics, including estimation in the presence of missing data and growth curve models for longitudinal data. These topics both require a
thorough understanding of models for means and intercepts, which are usually covered in week 3.

What sort of a background is required for this course? At the very least, individuals should have taken the I.C.P.S.R. Regression Analysis II workshop or its equivalent (note that this is a second level graduate regression course), or its equivalent. A thorough familiarity with regression models is absolutely essential. Taking the two courses simultaneously (this course and the Regression Analysis II: Linear Models course) is not recommended. A good understanding of the rudiments of matrix algebra is also important. While I.C.P.S.R. offers a set of Matrix Algebra Lectures early in the second session and while these lectures can help participants improve their matrix skills (indeed, beyond what is needed for the General Structural Equations course), the option of taking this course without any prior matrix algebra training should be considered only by those individuals who are not taking the course for formal university credit – and even at that, caution is appropriate. Some exposure to factor analysis will be helpful, since there are distinct parallels between some aspects of SEM modeling and factor analysis, but should not be considered essential.

Some form of an introduction to simultaneous equations and causal models is recommended but is not an absolute requirement. This year, the Simultaneous Equation Models workshop is offered during the second term; participants may wish to consider taking this course at the same time.

Required and Recommended Readings:

The major textbook for the class is a manuscript An Introduction to Structural Equation Models for Latent Variables, that I have prepared for the class. Chapters of this text will be available to I.C.P.S.R. participants for the cost of photocopying and should be considered essential for the course. (Instructions for obtaining these will be announced in the first class). It is required.

While this manuscript covers most of the material dealt with in the course, participants may wish to purchase copies of an additional text, since the ability to “triangulate” explanations is sometimes helpful in learning new techniques. Some copies of the following may be obtained at the bookstore (but participants could consider sharing a copy with a fellow participant or borrowing one of the multiple copies from the ICPSR library as required). These texts should be considered recommended.


2. David Kaplan, Structural Equation Modeling. Sage, 2000. This text is frustratingly terse at points and is thus not necessarily a good text for individuals who are not already familiar with latent variable structural equation models. It is in this sense better as a text to be read after a participant has finished the course (or at least most of the course). It contains useful treatments of some advanced topics: missing data, multilevel models, and latent growth curve models. (Only a small number of copies has been ordered for the bookstore; copies will also be available at the ICPSR library).
Supplementary Readings:

The following supplementary texts will also be useful during the course, but are not required:

1. An edited volume, called *Structural Equation Modeling: Concepts, Issues and Applications*, edited by Rick Hoyle (Sage, 1995). This text does not provide a thorough introduction to the area, but does contain some useful treatments of special topics and issues. (Only a small number of copies has been ordered for the bookstore)

2. Ken Bollen’s *Structural Equations with Latent Variables* (John Wiley, 1989) , which discusses models in matrix terms. (Only a small number of copies has been ordered for the bookstore).

Some more advanced texts are more appropriate for the material covered in the last few days of class and for “further study” after a participant has taken the course. These include:


None of these more advanced texts has been ordered for the bookstore, but copies should be available in the ICPSR library.

In the past, we have sometimes ordered copies of software manuals. Because software vendors typically do not have liberal “returns” policies (if the bookstore does not sell copies, it cannot return them for a refund), we have not ordered any this year. Class participants should not require manuals for purposes of the course, since handouts on the use of the main software programs (AMOS, LISREL, MPlus, EQS) will be provided. If there is class interest, handouts for some lesser-used software programs (the CALIS procedure in SAS and the third-party GLLAMM module in STATA) will be provided. For AMOS, LISREL, Mplus and EQS, there will be  multiple copies of the manuals both in the library and in the computer lab(s). This year,
we will be placing slightly more emphasis on EQS and MPlus than on other software, but special
lab sessions for AMOS will also be provided.

Instructions on the use of different computer software programs will be available in special class
handouts dealing with each of the software programs we will be using in the course.

**Assignments and Exercises**

Most participants in this workshop do not attend for the purposes of obtaining formal course
credit. For non-credit participants, it is important to complete as many of the computer
exercises as possible; without practical experience working with software and writing up
“results,” participants are not likely to be able to conduct research of their own using the
methods discussed in the course. There will be six computer exercises throughout the course
(although the completion of the sixth is not necessarily expected as it will come late in the
course). Individuals taking the course for credit (or to receive a letter with a grade) will also be
asked to write 2 take home tests and one “in class” test as well as writing a formal “write up” of
research results. The tests are not particularly important for participants who do not require a
course grade.

It is important that individuals who require a grade at the end of the course (taking the course for
formal credit or would like ICPSR to write a letter indicating the grade that was received)
identify themselves at the beginning of the course or mark “credit” or “grade” on their
assignments. Assignments submitted by non-credit participants not requiring a grade are
returned with comments and suggestions, but not with a grade.

Participants requiring a grade should ask for a copy of a “Grade Information” sheet which
provides further information on the computation of formal grades for the course.

In the past, participants have asked if it would be possible to substitute any course requirements
for a “major project” involving data that they are interested in working on. Unfortunately, the
brevity of the summer program makes this alternative form impossible.

**Detailed Topic Outline and Reading List:**

The main reading for the course comes from the (Baer) manuscript to be distributed by
I.C.P.S.R. (for the cost of photocopying), and from Schumacker and Lomax. Please pay careful
attention to the distinction between readings labeled **Required, Recommended or Optional**, on
one hand, and readings marked **Further Reading**. The first three types refer to readings that
should be read as their contents are discussed in class (preferably before). Most of the articles
and book chapters listed in this outline are, however, listed as **Further Reading**. These represent
*continuations* of the material covered in class and will not necessarily be dealt with in class
itself. In other words, it will not be necessary to read these materials before or immediately after
the class covering the topic in question. Participants may, as time permits, read them when it is
convenient.
Many, but certainly not all, of the readings involving journal articles, are available at the Helen Newberry Library. Most are also available electronically at the University of Michigan Library. If there is a particular journal article that you cannot obtain online and cannot locate in the Helen Newberry Library, please conduct us and we will try to make a copy available for loan.

Commonly used journals are referred to in short form, as follows:

“SEM” – Structural Equation Modeling  
“SMR” – Sociological Methods and Research  
“MBR” – Multivariate Behavioral Research  
“PM” – Psychological Methods

About the coverage of topics:

Each topic area will normally take approximately two to three hours to cover, but some may take considerably more time and some may possibly take less.

1. An Overview

Required reading: Baer, chapter 1; Schumacker and Lomax, chapter 1.

Topics: Linear models for path/regression analysis; conceptualizing latent variables; structural equation models for latent variables; measurement error and its implications.

Optional: First chapter (Hoyle) in Hoyle Kaplan, chapter 1.

2. Software Overview; Scalar Programming and Model Specification

A special handout will be provided

We will discuss the advantages and disadvantages of various software programs, to assist participants in making a choice of the “primary” software package they would like to work with. There will be a brief demonstration of AMOS, but the class will focus on programs which require the listing of equations in scalar form: MPlus, EQS, and the SIMPLIS interface of LISREL. If time permits and if there is enough interest, we will also look at SAS-CALIS and GLLAMM in STATA.

A small amount of time will be devoted to the use of the following packages for scalar model notation:

- LISREL (SIMPLIS interface)  
- MPlus  
- EQS

The following will be covered fairly tersely and only if there is class interest:
- The CALIS procedure in SAS
- The GLLAMM procedure in STATA

[Refer to Schumaker and Lomax, chapter 8]

**The second part of one of the classes this week (probably class #3) will be devoted to a hands-on introductory lab emphasizing software involving scalar programming (EQS, MPlus, LISREL- SIMPLIS). This lab will be roughly 2 hours, but, for participants who cannot attend past 5pm, it will be designed as a set of “self-instructional” steps.

**For participants who would like to work with AMOS software, there will be a special lab outside of regular class hours, probably on the Thursday of the first week. Self-instructional handouts will also be available.

3. Covariance Algebra for Latent Variable Models; Identification

Required reading:Baer, chapters 2 & 3; Schumacker and Lomax chapters 2,3,6

Kaplan, chapter 2 (to p. 24); also pp. 48-50*  
*Kaplan’s chapters all use matrix notation, which is not discussed until later

Topics: The basics; systems of equations; applications to path analysis models; reproduced covariances/correlations; direct and indirect effects; applications to factor models; under-identification and its implications; over-identification and its uses; establishing testable hypotheses; identification in factor models; identification in non-recursive causal models

Computer assignment #1 given out approximately here. Approximate due date: Week 2, Monday.

Further reading:  
Kenny, chapters 6, 7, 8.  
Bollen, chapter 7  

*more advanced; may not necessarily be discussed in class or may be discussed very briefly

4. Scaling and Interpretation Issues; Model Fit and Model Improvement

Required reading: Baer, chapter 4

Recommended:
  Schumaker and Lomax, chapter 7;
  L. Hu and Peter Bentler, "Evaluating Model Fit", chapter 5, pp. 76-99 in Hoyle, Structural Equation Modeling.

Optional: Schumaker and Lomax, chapters 4 & 5.
  Kaplan, pp. 34-39*
  *explanation is in matrix terms

Topics: Establishing a metric for latent variables; constructing linear composites; fixed and free parameters in models; covariances among latent variables; variances of latent variables; standardized solutions; mixing (single-indicator) manifest and (multiple-indicator) latent variables; reproduced vs. empirical covariance matrices; the chi-square test for model fit; incremental chi-square tests; some goodness of fit indices; Lagrange Multiplier tests, modification indices, Wald tests;

Further Reading:

W. Bielby, Arbitrary Metrics in Multiple Indicator Models, SMR, 15(1), 1986, pp. 3-23
Bollen, chapter 8, pp. 349-355.
R. MacCallum, Model Specification, chapter 2 in Hoyle.
Blair Wheaton, Assessment of Fit of Overidentified Models, SMR, 16(1), 1987, pp. 118-154

The first take-home test will be distributed approximately here. Due on Thursday of week 2.

5. Estimation; More on Fit Indices

Required reading: Baer, chapter 5

Optional: Loehlin, chapter 2; chapter 7 (pp. 195-204)
  Kaplan, chapter 2, pp.. 24-34; chapter 6.
Topics: testable and non-testable hypotheses; exploratory modification of models; equality constraints; fit indices: sample size bias; parsimony; typical values in sparse vs. dense models; relationship to model replication issues; equality constraints; linear constraints

Further reading:

L. Hu and P. Bentler, Evaluating Model Fit, chapter 5 in Hoyle
Ding, L., Velicer, W., & Harlow, L. Effects of Estimation Methods, Number of Indicators Per Factor, and Improper Solutions on Structural Equation Modeling Fit Indices., SEM, 2(2), 1995, 119-144.
Ken Bollen, A New Incremental Fit Index, SMR, 17(3), 1989, pp. 303-316.
Jackson, Dennis, “Revisiting Sample Size and Number of Parameter Estimates: Some Support for the N:q Hypothesis,” SEM, 10(1), 2003, 128-141.

6. Simultaneous Analysis in Multiple Groups

Required Reading: Baer, chapter 8

Optional: Bollen, chapter 8, pp. 355-365
Schumaker and Lomax, chapter 10.3.
Kaplan, chapter 4
Topics: Introduction: replicating models across groups; across-group parameter constraints; testing for measurement equivalency; testing for equivalency of causal (structural equation) effects; comparisons with analysis of covariance designs

Further reading:


Computer exercise #2 approximately here. Tent. due date: Friday, week 2

7. Programming multiple group models

Handouts to be provided

Class: one hour
Lab: two hours

A special lab on multiple-group models will be run in the last hour of the regular class period. This will be a 2-hour lab, but for participants who must leave at 5pm, self-guided instructions will be provided so they can work on their own outside of any formal class/lab period. The lab will include “setup” details – instructions as to how data can be brought in from common stats packages into SEM programs for purposes of multiple-group analysis.

8. Matrix Algebra for Path and Factor Models; LISREL model notation

Required Reading: Baer, chapter 7

Optional Reading:
Bollen, Appendix A and chapter 2 or
Hayduk, Structural Equation Models with LISREL, chapter 3, pp. 56-77 and chapter 5, pp. 132-142.

Topics: covariance structure algebra in matrix terms; reproduced covariances in matrix terms. LISREL model notation for structural equations involving observed variables; LISREL model notation for confirmatory factor models; LISREL model notation for structural equations involving latent variables; the exogenous-endogenous distinction; comparisons with scalar models

** For participants who would like to learn more about programming LISREL, some examples along with a self-instructional guide will be provided.
9. Distributional Assumptions, the ADF Estimator, Robust Test Statistics, Bootstrapping, Item Parcelling

Recommended: Bollen, pp. 415-432
Kaplan, chapter 5 (to page 87)

Topics: Data screening; data transformations for continuous data; discrete and coarsely-categorized variables; dichotomous variables (as X-variables; as indicators); robustness of ML estimator; “robust” statistics; the ADF estimator; polychoric correlations for ordinal data

Further Reading:
Browne, M. Asymptotically Distribution-Free Methods for the Analysis of Covariance Structures, *British Journal of Mathematical and Statistical Psychology*, 37, 1984, pp. 62-83. (mathematically intense and will not be discussed in full detail, but provides the basis for what has come to be known as the ADF estimator)
S. West, J. Finch and P. Curran, Structural Equation Models with Nonnormal Variables: Problems and Remedies, chapter 4 in Hoyle.
S. Green et al, Effect of the Number of Scale Points on Chi-Square Fit Indices in Confirmatory Factor Analysis, SEM, 4(2), 1997, 108-120.(see also Yung and Bentler under "Bootstrapping Approaches")
Chan, W., Yung, Y., & Bentler, P. A Note on Using an Unbiased Weight Matrix in the ADF Test Statistic MBR, 30(4), 1995, 453-459.

Further Reading on Bootstrapping:

Yung, Y.F. and P. Bentler, Bootstrapping Techniques in the Analysis of Mean and Covariance Structures. In M & S.

Further Reading, Item Parcelling:


Computer assignment #3: Models for non-normally distributed data. Tent. due date: Tuesday, week 3

**Tentatively, there will be an extra outside-of-class lab session dealing with approaches to non-normal data in LISREL/SIMPLIS, EQS, MPlus and possibly AMOS.

10. Mean and Intercept Models

Required Reading: Baer, chapter 9.

Optional Reading: Loehlin, pp. 204-210.
Kaplan, pp. 68-70.

Topics: Adding intercepts to latent variable models; factor mean comparisons; mean comparisons in structural equation models
Further Reading:

Bollen, chapter 7, pp. 306-311
Bollen, chapter 8, pp. 365-368
*more advanced treatment of topic to be covered only if time permits

Computer exercise #4: Multiple Group Models for means and intercepts
Tentatively due: Thursday, week 3

A special lab on models for means and intercepts will be held outside the regular class, probably on Monday of week 3. A self-guiding instructional handout will be available to those who are unable to attend this session.

11. Problems and Issues

**Depending on the progress of the class with respect to other topics, the topics below might be covered in a fairly terse fashion, with class participants encouraged to read the additional readings below to supplement the material covered in class.

Optional: Bollen, chapter 7, pp. 281-286.
Kaplan, pp. 79-80.

Topics: Sample size and goodness of fit; improper parameter estimates; collinearity; missing data; identification in complex model; inequality constraints; categorical exogenous variables; weighting and stratified samples [if time permits: power in significance tests ##]

Further reading on improper solutions:
Further readings on statistical power

Further readings on sample weights:

Further Reading on Sample Size and Small Samples:
Jackson, D.  The Effect of the Number of Observations per Parameter in Misspecified Confirmatory Factor Analytic Models, SEM, 14(1), 2007, 48-76.
Herzog, Walter and Anne Boomsa, “Small-Sample Robust Estimators of Noncentrality-Based and Incremental Model Fit” SEM 16(1), 2009, 1-27.

Write-up assignment; tentative due date Monday, week 4.

12. Review and Discussion

1. A critical discussion of literature from various disciplines, including readings suggested by class members.
2. Review of key concepts.
3. Outline: how to present SEM model results (what to discuss, etc.)

** In the event that previous topics take more time than anticipated, this component might be truncated or eliminated.
13. Missing Data in SEM Models

*Required, if available. Baer manuscript extra chapter.*

**Recommended:**
Bollen, pp. 369-376.
Kaplan, chapter 5, pp. 87-96.

**Further Reading:**
J. Arbuckle, Full Information Estimation in the Presence of Incomplete Data, chapter 10 in M & S.
Gold, Michael, Peter Bentler and Kevin Kim, “A Comparison of Maximum-Likelihood and Asymptotically Distribution-Free Methods of Treating Incomplete Nonnormal Data,” SEM, 10(1), 2003, 47-79
Graham, John, “Adding Missing-Data-Relevant Variables to FIML-Based Structural Equation Models,” SEM, 10(1), 2003, 80-100.

Computer exercise #5: Missing Data
Tentative due date: Tuesday, last week of class.

14. In-Class Exam (credit participants)
Does not cover missing data.
Or Special Lab: Software approaches to missing data (non-credit participants)

15. Models for Panel Data

Topics: Direct and indirect effects in causal models; non-recursive models contemporaneous and lagged effects in a two-wave panel model; correlated measurement error in panel models; relationship between SEM approaches and ARMA time-series models


Further Reading:

Karl Jöreskog and Dag Sörbom, Advances in Factor Analysis and Structural Equation Models, chapters 5 (Statistical Methods for Analysis of Longitudinal Data) and 6 (Detection of Correlated Errors in Longitudinal Data).
Lawrence Mayer and Steven Carrol, Testing for Lagged, Cotemporal and Total Dependence in Cross-Lagged Panel Analysis, SMR, 16(2), 1987, pp. 187-217
J. Willett and A. Sayer, Cross-Domain Analysis of Change Over Time: Combining Growth Modeling and Covariance Structure Analysis, chapter 5 in M & S.

16. Growth Curve Models

Further Reading:

Computer Exercise #6: A Longitudinal Data Model. Due: Last day of class.
17. Polynomials, Interactions and Non-Linear Models

*Read:* Bollen, chapter 9, pp. 403-415
L. Hayduk, *Structural Equation Modeling with LISREL*, chapter 7, pp. 219-244.

*Further Reading:*

R. Schumacker and G. Marcoulides (eds.) *Interaction and Nonlinear*. (Entire text, but especially chapters by Rigdon, Schumacker and Wotechke [ch. 1], Jonsson [ch. 2], Bollen and Paxton [ch. 6], Joreskog [ch. 11]

*Take home exam #2 distributed; due date: last day of class.*

18. Latent Class Analysis; Mixture Models

*Further Reading:*

B. Muthen, Goodness of Fit with Categorical and Other Nonnormal Variables, chapter 9 in Bollen and Long.


Allan McCutcheon, Latent Class Analysis. Sage Quantitative Applications Paper No. 64 (1987)


19. Multilevel Analysis in Structural Equation Modeling

Further Reading:


Patrick Curran “Have Multilevel Models Been Structural Equation Models All Along?” MBR, 39(4), 2003, 529-569
