Introduction

This lecture series will provide a background in basic matrix algebra skills for participants in ICPSR Summer Program workshops. We will spend the next two weeks developing these skills. The lectures are designed to serve both as a refresher for those previously exposed to matrix algebra and as a quick (and necessarily limited) introduction for those new to the material.

Proficiency in basic mathematics (e.g., elementary algebra and summation notation) is assumed.

While there is no text for this series of lectures, a good treatment of basic matrix algebra can be found (generally in an appendix...) in most Econometric and Regression textbooks.

There is no graded material, nor are any grades given, for this lecture series. I will distribute a pencil-and-paper worksheet and answer key, along with a few other handouts.

The first lecture will be Tuesday, July 21, 2015. The final lecture will be Friday, July 31, 2015.

Matrix Algebra and Regression

While knowing and being comfortable with matrix algebra is extremely important for participants who are currently taking a course involving OLS Regression, this lecture series is not merely a “lab” or “supplement” to any particular workshop. We will, instead, approach Matrix Algebra from a more broad and generalizable perspective.

Notice, however, that the last topic we will consider is “Using Matrices to Represent Linear Regression Models.” This is but one example of an application of the Matrix Algebra material we will cover in this lecture series.

Conclusion

Learning the material in these lectures will require a substantial amount of effort on your part—but that is why you are here! The payoff will be well worth that effort.

It is an honor to be your instructor for this series of lectures.

The next three pages contain an outline of the Lecture Topics.
Lecture Topics

I. Introduction to Matrices
   A. Definition
      1. As a Set of Numbers (Scalars)
      2. Dimension (Order) of a Matrix
      3. Special Case: The 1 x 1 Matrix
      4. Element Notation
   B. Equality of Matrices
   C. Row Matrix (Vector)
   D. Column Matrix (Vector)
   E. Zero Matrix
   F. Transpose of a Matrix
   G. Square Matrices
      1. Definition
      2. Diagonal
      3. Trace
      4. Symmetric Matrix
      5. Triangular Matrix
         a. Upper
         b. Lower
      6. Diagonal Matrix
      7. Scalar Matrix
      8. Identity Matrix
         a. Special Notation for the Identity Matrix
         b. Mathematical Definition of the Identity Matrix
   H. Partitioned (Block) Matrix

II. Matrix Addition and Subtraction
   A. Conformability Conditions
   B. Matrix Addition
      1. Method
      2. Mathematical Definition
   C. Matrix Subtraction
      1. Method
      2. Matrix Negation
      3. Mathematical Definition
   D. Properties of Matrix Addition and Subtraction
   E. Adding and Subtracting Partitioned Matrices

III. Multiplication of a Scalar and a Matrix
   A. Method
   B. Mathematical Definition of Scalar and Matrix Multiplication
   C. Properties of Scalar and Matrix Multiplication
IV. Matrix Multiplication
   A. Left- (Pre-) and Right- (Post-) Multiplication
   B. Conformability Conditions
      1. Requirements (The “Inner” Dimensions)
      2. The Product Matrix (The “Outer” Dimensions)
      3. Conformability and Lack of Commutativity (i.e., “Order Matters”)
      4. Multiplication of a Matrix and its Transpose
      5. Multiplication Involving a 1x1 Matrix
      6. Inner Product
      7. Conformability and Square Matrices
   C. Method
      1. How to Do It
         a. The “Striped Shirt” Explanation
         b. The “Diving Board” Explanation
         c. The “Point-and-Click” Explanation
      2. Lots of Examples of Matrix Multiplication Using Real Numbers
         a. General Examples
         b. Square Matrices: Lack of Commutativity, Even if Conformable (i.e., “Order Still Matters”)
            c. Row and Column Vectors: Inner Product and Trace
            d. A Matrix and its Transpose: Symmetry and Equal Traces
            e. Diagonal Matrices
            f. Multiplication by a Zero Matrix
      3. Formal Mathematical Definition
   D. More on the Identity Matrix
      1. Definition of a Multiplicative Identity
         a. Scalar
         b. Matrix
      2. Multiplication Involving the Identity Matrix
   E. Powers of a Matrix
      1. Integer Powers
      2. Fractional Powers
      3. Idempotent Matrix
   F. Properties of Matrix Multiplication (Including Transpose Matrices)
   G. Representing Systems of Linear Equations with Matrices
      1. The Coefficient Matrix
      2. The Variable Matrix
      3. The Solution Matrix
      4. Matrix Multiplication and Linear Equations
   H. Silly (and Memorable) Examples
      1. Tables and Stools
      2. Suppliers and Cost
      3. Dewey, Cheatem, and Howe

V. The Kronecker (a.k.a. Tensor) Product: \( A \otimes B \)
VI. The Inverse of a Matrix
   A. Definition of a Multiplicative Inverse
      1. Scalar
      2. Matrix
   B. Notation for the Inverse Matrix
   C. Matrix Multiplication Involving an Inverse Matrix
   D. Singular and Nonsingular Matrices
      1. Definitions
      2. Some Necessary Conditions for Nonsingularity
   E. Inverse of a Diagonal Matrix
   F. Properties of Inverse Matrices
   G. Computing an Inverse Matrix: A Very Quick Overview

VII. Determinant of a Matrix
   A. Condition for Existence
   B. Notation
   C. Computing the Determinant
      1. 2x2 Matrix
      2. 3x3 Matrix
   D. Determinants and (Non)Singularity
   E. Another Method of Computing an Inverse Matrix: A Very Quick Overview
      1. Minor of an Element
      2. Cofactor (Signed Minor)
      3. Cofactor Matrix
      4. Adjoint Matrix

VIII. Rank of a Matrix
   A. Definition of Linear (In)Dependence
   B. Definition of Rank
   C. Rank of a Square Matrix
      1. Rank and Determinant
      2. Rank and (Non)Singularity
   D. Definition of a Positive-Definite Matrix

IX. Eigenvalues and Eigenvectors
   A. Calculations
   B. Uses

X. Variance-Covariance Matrices

XI. Using Matrices to Represent Linear Regression Models
   A. The Multiple OLS Regression Model
   B. Deriving the Estimation of the “B” Matrix
   C. Examples of Representing Data and Manipulations in Matrix Format
   D. Rank, Singularity, Multicollinearity, and Bombs
   E. A Proof that the Matrix Derivation of “B” Minimizes the SSE